What is algebra?
(with strong apinions)

whats so special about ^2?

whats so special about = ?

What is algebra?
- Study of equations!
- Solving equations

Really why is algebra? → - Solving equations

Ex $x^2 - y^2 = 0$ Variables: x, y

$\frac{\partial w}{\partial x^2} - \frac{\partial w}{\partial y^2} = 0$: $^2, -, 0$

$m^2 - n^2 \equiv 0 \pmod 7$: $=$

Agebraicly these are the same!

In this example we can factor

$(x-y)(x+y)=0$, $\left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) = 0$, $(n-m)(n+m) \equiv 0$

Factoring only require distributivity!

Big Unspoken Rule/Idea in algebra

"Organize algebraic structures by operators and laws"

→ Algebra comes from specific examples but an "algebraist" doesn't care about examples or specific implementations of operators, equality, constants

→ Algebra cares only about the symmatric structures of equations and the laws that govern them

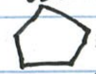
An Algebraic structure has

		operators	laws
Ex	Group	$\{\square \cdot \square, \square^{-1}, 1\}$	$(x \cdot y) \cdot z = x \cdot (y \cdot z)$, $x \cdot x^{-1} = 1 = x^{-1} x$, $1 \cdot x = x = x \cdot 1$

where do these operations come from? Symmetry! (Equations on symmetry)

An example of a Group will have (1) Constants : G usually a set

(2) Implementation of operators : usually functions $\cdot : G \times G \to G$

Ex Symmetries of regular , $\square \cdot \square$ = composition, $\square^{-1}$ = undo, 1 = do nothing

Ex Symmetries of $\mathbb{R}^2$ $GL_n(\mathbb{R})$ general linear group

		operators	laws
Ex	unital Ring	$\{+, -, 0, \cdot, 1\}$	

Homomorphisms (alg structure preserving maps)

↳ operators

Ex Group homomorphism

$f : (G, \{\cdot, \square^{-1}, 1_G\}) \longrightarrow (H, \{*, \square^{-1}, 1_H\})$

$f(g \cdot h) = f(g) * f(h)$, $f(g^{-1}) = f(g)^{-1}$, $f(1_G) = 1_H$

Noether's Isomorphism Theorem's (First Isomorphism theorem)

Congruence on level of Sets (Resonance)

$a \sim b$ on $X \longmapsto$ equivalance classes $\{[a]_\sim = \{a \sim b \mid b \in X\}\}$

Equivalance relations $\longleftrightarrow$ Partitions

$a \underset{f}{\sim} b \Leftrightarrow f(a) = f(b)$

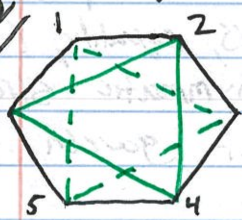
Function

$f : X \longrightarrow Y = \{[a]_\sim \mid a \in X\} = \{\text{Partitions}\}$

by $x \longmapsto [x]_\sim$ = the partition x is in.

OK Now with algebra Quotient

Congruence



G = Symm. of hexagon

$\sigma \sim w$ iff they do the same thing to the triangles

$Id \sim (1 \mapsto 5)(2 \mapsto 4)$ Don't Flip triangles

This preserves alg structure!

The operators $\{\circ, {}^{-1}, 1\}$

$\sigma \sim w$	$\sigma \sim w$
$\delta \sim \gamma$	$\sigma^{-1} \sim \sigma^{-1}$
$\sigma \circ \delta \sim w \circ \gamma$	$Id \sim w\sigma^{-1}$

2 blocks in Partition of equiv classes

$N = \{Id, (15)(24)\}$,

Symms that don't Flip the Δ's

$\sigma N = \{(12)(63)(45), (123456)\}$,

This also preserves structure!

$\{N, \sigma N\} =: G/N$ is group!

Note: N is normal Subgroup

Homomorphism

$f : G \longrightarrow G/N = \{N, \sigma N\}$, $g \mapsto gN$

but this could be any function (hom.)

$f : G \longrightarrow (\mathbb{R} - 0, \{\cdot, {}^{-1}, 1\})$

w/ property that whole blocks get mapped to the same element

$g \in N \longmapsto 1$; $g \in \sigma N \longmapsto -1$

ok an example of congluences in Rings Equiv relation

Common Idea in algebra (adding new laws)

→ Bake in new Plopecties maybe $xy \sim yx$

→ add new elements $0 \sim x^2+1$ makes x act as i

→ Changes granularity on "=" ⟶ "~"