The Colorado State Torus

Edition 1 March 2025

What is this?

This is the first edition of *The Colorado State Torus*, the first (as far as we know) graduate-student run newsletter for the mathematics department at CSU. The goal of this newsletter is multifaceted. First, we believe this newsletter will be a way to build community and communication among the students, faculty, and staff of the math department. Second, graduate school is difficult and long, and we hope that this project will allow both the editors and contributors to express their creativity in a low-stakes way. And finally, we at the CST believe that local journalism is important. This is just about the most hyperlocal newspaper you could ask for!

If you are interested in contributing to the CST in any way shape or form, please reach out! There are details at the end of the newsletter for contacting us, but you might already know us. This first edition has already been a great group effort and we look forward to continuing this project. See you next month!

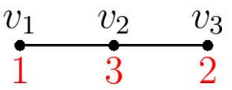
Joe Geisz and Ian Jorquera

The Stanley-Stembridge Conjecture Solved by Tatsuyuki Hikita!

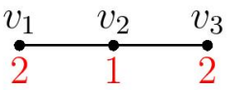
Maria Gillespie

Stanley defined the chromatic symmetric function of a vertex-labeled graph as follows: for every proper coloring of Γ with colors from $\{1, 2, 3, \dots\}$ (such that no two adjacent vertices have the same color), associate the monomial $\prod_i x_i^{m_i}$ where m_i is the multiplicity of the color i in the coloring. Then the sum of these monomials over all proper colorings is the chromatic symmetric function, written $\chi_\Gamma(\mathbf{x})$.

For example, consider the path graph below, with two colorings shown along with their associated monomials:



$x_1x_3x_2$



$x_2x_1x_2 = x_1x_2^2$

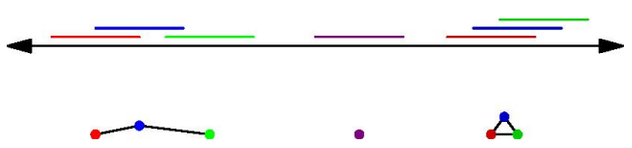
There are 6 ways to color it with any choice of three distinct colors, and one way to color it with two i's and one j for any i and j, and so the chromatic symmetric function looks like:

$$6x_1x_2x_3 + 6x_1x_2x_4 + \cdots + x_1^2x_2 + \dots = 4m_{(1,1,1)} + m_{(2,1)}$$

in terms of monomial symmetric functions. In terms of elementary symmetric functions, this equals

$$(x_1x_2+x_1x_3+\dots)(x_1+x_2+\dots)+3(x_1x_2x_3+\dots) = e_2e_1 + 3e_3$$

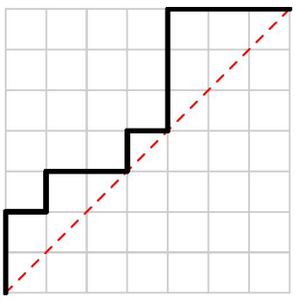
Stanley and Stembridge conjectured that for "nice enough" graphs, the chromatic symmetric function is e-positive, meaning that its coefficients in the elementary basis are nonnegative integers. Guay-Paquet reduced their conjecture (which was precisely about incomparability graphs of 3+1-free posets) to the case of unit interval graphs, which are graphs whose vertices $v_1, \dots, v_n$ correspond to unit length intervals ordered by their left endpoint on the real line, and such that there is an edge between two vertices if and only if the corresponding unit intervals overlap. For instance:



Unit interval graphs can be encoded combinatorially (up to isomorphism) by *Hessenberg functions*, which are weakly increasing functions $h : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ such that $h(i) \geq i$ for all i. The unit interval graph on $v_1, \ldots, v_n$ associated to h is the graph formed by connecting v_1 to all v_j with $j \leq h(1)$, then connecting v_2 to all v_j with $2 \leq j \leq h(2)$, and so on.

As an example, the unit interval graph above is associated to the Hessenberg function $h = (2, 3, 3, 4, 7, 7, 7)$, and we draw it as a Dyck path whose heights of the horizontal steps are $h(1), h(2), \ldots, h(n)$ from left to right:

$h = (2, 3, 3, 4, 7, 7, 7)$
$e = (0, 0, 0, 3, 4, 4, 5)$



In their recent proof, Hikita uses the *complementary Hessenberg function* $\mathbf{e} : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$ defined by $\mathbf{e}(i) = n - h(n + 1 - i)$. That is, the function $\mathbf{e}$ gives the number of squares above the Dyck path in each column from right to left, and in the above example, $\mathbf{e} = (0, 0, 0, 3, 4, 4, 5)$ with $n = 7$.

Problem of the Month

Problem provided by Sandra Nair:

Find all functions $f : \mathbb{Z} \to \mathbb{Z}$ such that $f(2x) + 2f(y) = f(f(x + y))$ for all $x, y \in \mathbb{Z}$

The answer will be included in next month's edition. Submit your solutions to the Colorado State Torus for bragging rights.

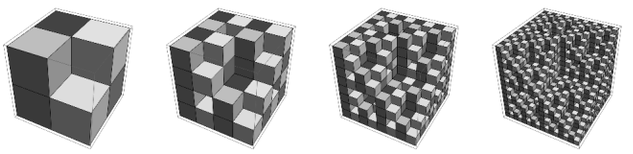
Looking for more Funding?

Are you a Colorado Resident? You may qualify for the first come, first serve Colorado Graduate Student Grant. Fill out your 2025-2026 FAFSA! You could receive up to $3,500 a semester from Colorado through the University.

Mathematical Curiosities

Joe Geisz

I wrote my Masters thesis back in 2020. I was working with a certain class of 3D fractals and was interested in what could be said about their topology. A specific example, that leads to an interesting result, is as follows. Imagine you have the closed unit cube, side length 1, in 3D space. Duplicate this cube 7 times, and fill in the 2x2 cube centered at the origin, except leave the octant with all positive coordinates empty. You now have a shape that looks like a cube with a chunk removed (The left most image in the figure). Now take this shape, duplicate it 7 times, and shrink these shapes to half their size. Then arange these shapes in the same way you did with the 7 cubes (The second image in the figure). If you can visualize in 3D, there is a void inside this shape, a "bubble" formed from enclosing the removed chunk of one cube on all sides by the faces of the other shapes.



Now imagine iterating this process repeatedly. At each step, how many voids are there in the shape? If you are intrigued, take some time to think about it and try to figure it out. At the 3rd iteration, you will have 7 voids that are simply due to duplicating the shape 7 times. But you will also have a big void in the middle again, and more voids from smashing the sides of the

smaller shapes together! In total, the 3rd iteration will have 17 voids. The 4th iteration will have 156. In order to work out the pattern, I built the 3rd iteration from Legos, which really helped to wrap my mind around the geometry.

Once you figure out the pattern, (spoiler alert) you can write out an explicit formula for the number of voids at each iteration. It is

$$\beta_2(n) = \frac{91}{24}7^n - \frac{27}{8}3^n + \frac{7}{12}$$

While this might make sense to all of you combinotorialists out there, I thought this was quite interesting. I personally don't think it's even obvious that this formula ought to be an integer for all n. I thought this was a fun toy problem I ran across during my own research, and thought others might find it interesting to contemplate as well. If you come across any "Mathematical Curiousites" in your research that you would like to share, please submit them to the editors.

Reflecting on Impostor Syndrome as a Fourth Year Graduate Student

Jerett Cherry

The research we do as graduate students is not the most difficult part of doing research. The most difficult part to overcome is feeling like we cannot do the research. Impostor syndrome is a term that we ought to be aware of since it explains the common feeling that you cannot do research and do not belong. If you haven't felt yet, you will feel before you graduate. As a fourth-year graduate student, I wanted to reflect on what I have personally learned about impostor syndrome. To do so, I will focus on two ways I felt it, and what steps helped me to overcome the feeling. My hope is that by naming impostor syndrome—and helping graduate students at earlier stages of their degree to name it—we can lessen the impact it has on us. The first step in fixing a problem is knowing what might cause it, so what does?

People who feel like they are not making progress feel like impostors. In graduate school, however, the metric of "progress" is measured differently than at any other time in our formal education. As your research begins, the frequent feedback from homework, exams, and presentations fades. When I began my research, I felt the lack of instant feedback affect the way I felt about my work. Where before I felt my progress reflected in grades, I then lacked a way to measure my progress, and it felt as though none was made. Of course, this was not true. Progress in graduate school is measured in terms of years, not semesters, not months, nor days. When I think that I am failing to make progress, I find that reflecting on where I am now versus my first year makes me recognize my capability as a researcher. So, when you feel like you are not making progress, don't compare today to yesterday, last week, or last month, but rather last year. This will show you your progress on the correct timescale.

People who doubt the value of their

research feel like impostors. Many of us believe—perhaps if only in the very back of our minds—that a PhD thesis should present groundbreaking work. To some extent, this is true. Yet don't be fooled, you can break ground and not find much buried there besides dirt. The true measure of the quality of our graduate projects ought to be "Do you get your degree?", not "Does your thesis start an academic revolution?". Doubt can creep in when you compare yourself to a senior researcher, for example. It seems that the quality of your work is worse than theirs. It is not so. They are years down the road from you. Whenever I felt that the quality of my work was poor, something that helped me was to share my ideas with peers. Often, they found it interesting! Listening to their ideas also helped me: attend PhD defenses, preliminary exams, and part B's. Doing so will better set the bounds by which you should measure your work's quality. It did for me.

No matter how many times you overcome impostor syndrome, you must remain cognizant that over the span of years it can creep back in. Since grad school is long, just take the time to reflect on growth over years. It can help you recognize your worth as a researcher, student, and person.

United Campus Workers

Colorado State University is the newest chapter of United Campus Workers Colorado! Workers voted to affiliate with CWA Local 7799 as a wall-to-wall union, uniting all job classes including Admin Professionals, Graduate Workers, Faculty (non-tenure track, tenure track, and adjuncts), Undergraduate Workers, Postdocs, and all those working at Colorado State University. Together we can win raises, benefits, and improve working conditions for all CSU workers. Join us! ucwcolorado.org/csu

Seminars

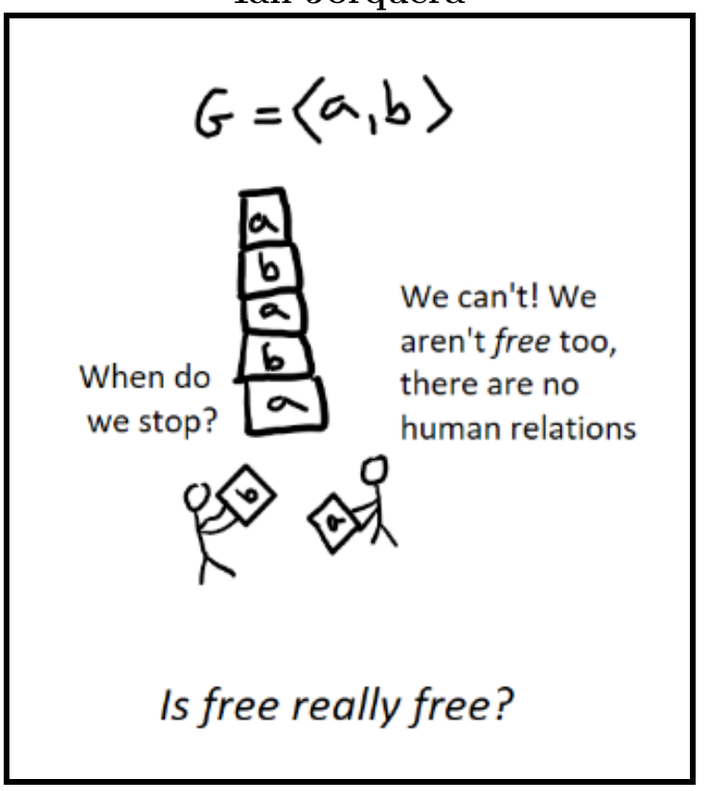
- *Applied Category Theory Seminar (ACTS)*
  Weber 237, Thursdays 4-5pm
  contact: nathaniel.collins@colostate.edu
- *Codes and Expansions (CodEx)*
  Online only, Tuesdays 11am
  math.colostate.edu/∼king/codex
- *Greenslopes (Graduate Students Only)*
  Weber 201, Thursdays 11am
  contact: ashley.armbruster@colostate.edu
- *IDA Seminar*
  Weber 223, Thursdays 3-4pm
  contact: jennifer.l.mueller@colostate.edu
- *Math Ed Seminar*
  Weber 15, Tuesdays 11-12am
  contact: hortensia.soto@colostate.edu
- *MMaRgs/Fragment*
  Weber 201, Thursdays 3-4pm
  contact: mark.shoemaker@colostate.edu
- *Number Theory Lab*
  Weber 223, Thursdays 12pm
  contact: cigole.thomas@colostate.edu
- *Pattern Analysis Lab (PAL)*
  Weber 11, Wednesdays 3-4pm
  contact: michael.kirby@colostate.edu
- *RMAC*
  Weber 223, Fridays 4-6pm
  contact: james.wilson@colostate.edu
- *Matemáticas*
  TBD
  contact: jake.kettinger@colostate.edu

Foto del Mes

This month's photo is from Burrito Day at the new grad space! Thanks to Ignacio for the photo.

March Comic

Ian Jorquera



Sports

This semester we are proud to congratulate our inner-tube water polo team, the Sitting Ducks, on their 3 victories so far this season! From all of us at the Torus, we wish you a strong end of the season.

Upcoming Intramural Sports Leagues

- One Day Ultimate Frisbee Tournament. Registration Deadline: April 14th

Call to Contribute

We hope you have enjoyed the first edition of The Colorado State Torus. Do you have any ideas for articles, opinion pieces, or new sections? Do you have an error to point out in a seminar time? Just want to give us a piece of your mind? We would love to hear from you! The CST is made possible through contributions from many people across the department. Send any suggestions, announcements, ads, or content to **MATH_ColoradoStateTorus@mail.colostate.edu**. The editorial board will review any ideas and hopefully include them in an upcoming issue. Thank you!

Greenslopes

Greenslopes is a graduate student seminar where grad students can practice talks, discuss interesting/fun ideas, and share their research. The rest of semester has a full line-up of amazing missions (talks); we would love to see you at lift off on Thursdays at 11 am in Weber 201!

Association for Women in Mathematics

The Association for Women in Mathematics at Colorado State University meets once a month on the first Monday or Tuesday of each month. AWM is open to all and often provides snacks, so drop in on one of our meetings! Past AWM events have included info sessions on undergraduate research, panels on motherhood in academia, and craft nights. AWM also runs a mentoring program each semester pairing undergraduates, graduate students, and faculty, so be on the lookout for the mentor/mentee sign-ups next fall! For more information and to sign up for our mailing list visit: mathematics.colostate.edu/awm/

AWM

ASSOCIATION FOR
WOMEN IN MATHEMATICS

March Issue Crossword

Kyle Salois

ACROSS

1 Trig. function
4 Actor Scott or Driver
8 End of a college address?
9 Exploding stars
11 Citrusy treat
14 Four-line rhyme scheme
15 Student org. in this department
16 Prosciutto, e.g.
19 German automaker
20 Up-and-coming newsletter
23 Quote from Homer
24 Chess ranking system
25 Inland Asian sea
27 Quench
31 Major vanilla exporter
34 "-phile" meaning
35 Reagan Airport, on tickets
36 Take a hike
37 Trig. function

DOWN

1 Emmy winner Ward
2 Lightbulb, metaphorically
3 Insensitive
4 ___ Arbor, Mich.
5 A.G.'s domain
6 Mountain and University, for two
7 City affected by the Palisades fire
10 Poetry contests
12 Honolulu's island
13 Texter's "Here I come!"
17 Excelled, in Gen Z slang
18 Chem. unit
20 What you might find near 12 down
21 Groundhog's omen
22 Rising pickleball star Flaxman
23 Beaver's bungalow
26 Obsidian, once
28 "Highway to Hell" band
29 Folded fast food
30 Taylor Swift's ___ tour
32 Shower goop
33 "Raiders of the Lost ___"