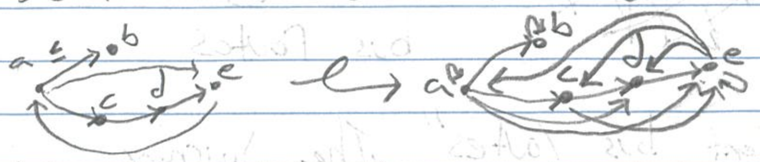
Last time: Pre-orders as an example of a category

Ex bus routes as a transitive (refl) closure



with so sneaky trickery on equality

$a \to e = a \to c \to d \to e$

ie don't care about "efficiency" of path

Galois connection as a way to take about "common data"

$$\begin{tikzcd} (A, \leq_A) \arrow[r, bend right, "f"'] & (B, \leq_B) \arrow[l, bend right, "g"'] \end{tikzcd}$$

$$\begin{array}{ccc} g(b) & \longleftarrow & b \\ \vee\!\!/\!\!\ast & & \vee\!\!/ \\ a & \longrightarrow & f(a) \end{array}$$

possibly flipped "twisted"! "op"

This is hard to work with but $a \leq_A g(f(a))$
and $g(f(g(f(a)))) = g(f(a))$
$g(f(-))$ is closure ie tells you what are the most important points in common data

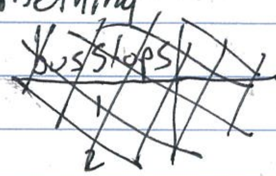
A real world problem: 40% of IT budges is "data integration"

Data is created ad hoc ie for a specific purpose and this is ok!

Sometimes!

Databases store information about something

Data: ~~Bus stops~~



Efficient bus Routes
vs
bus routes

In "efficient bus routes" the journey $a \to c \to d \to e$ and direct path $a \to e$ are considered differently. in "bus routes" they are equal

→ They tell us about the same data!
We will express this w/ a functor!

→ this can give us a model for understanding data migration and if it is possible "or dataset communication

→ through adjoint pairs (generalization of galois connection) telling us about how two database do/don't preserves the same "data"

Two data base can be "migrated" if

$${}_xX_Y \cong {}_xY_Y$$

The way data from either X, Y interact with database X is equiv to the way data interacts with database Y

Key Idea: databases are Ad hoc and that is OK!

What IS a Cat and why are they databases? or why are databases examples?

A category is Composition (a binary operator) with Rules for when you can

What are we composing?

1) morphisms: $\mathcal{C}$ $\quad \cdot \xrightarrow{f} \cdot \xrightarrow{g} \cdot$ when $tgt(f) = src(g)$

where $f, g \in \mathcal{C}$

so $\mathcal{C} \times \mathcal{C} \to \mathcal{C}$ only defined when

$(f, g) \mapsto g \circ f$ when

1c) "objects" one can consider the src and targets

1b) associativity of composition

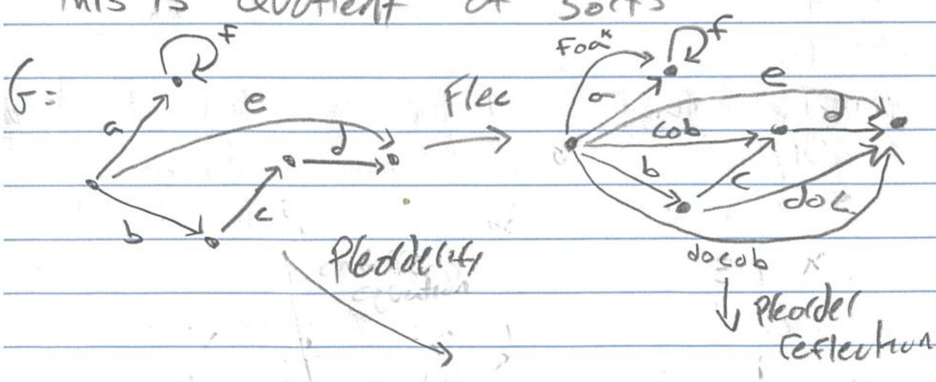
1d) Identity morphism for each object

$1_B \circ f = f = f \circ 1_B$

Ex Graph $G \rightsquigarrow$ transitive/refl closure

possibly doing trickery with equality

This is Quotient of sorts



Category Set (of functions)
morphism : functions $\{f : A \to B\}$
functors know about objects
Blah Blah Blah

Category of grp (of group homs)

Important terminology
The morphism that are composed with morphism f (ie $\_ \circ f$) are G^f, precomposed $_gG$ and both pre-post-composed $_gG^f$
let f, g be identities then
hom(tgt(f), src(g)) = $_gG^f$

Isomorphism is a thing

Databases

don't limit yourself with reality

Planets	Volume
Mercury	2
B	1000
Earth	42
tatooine	5

Geese	instrument	Real no
Cameron	Vocal/piano	0
Emily	guitar	1
Dominic	bass	π
Max	drums	i
Sam	keytar	

These are objects we may want to work wi

adding a new column represents a function

Composition : applying multiple function (multiple tables)

eese	instrument

musical instruments	
vocals	
piano	
holophoner	

Category of a database

Functors (homomorphisms of categories)
(play the role of monotone maps)
$F: C \longrightarrow D$ s.t preserve composition
of morphisms and identity

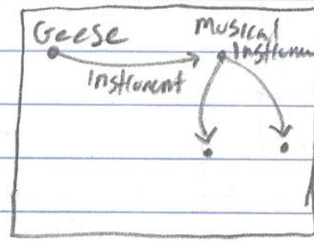
$$\underset{C}{F(f\circ g)} = \underset{D}{F(f)\circ F(g)}$$

Ex: Bus routes $\underset{\text{pick a route}}{\overset{\text{collapse routes}}{\rightleftarrows}}$ efficient bus routes

Ex Cat "from primordial ooze"

Natural transformations

Actually, some formalization
database as Schema



You may also impose composition rules related to equality

This is a graph so by transitive/refl closure and restrictions on equality you have A category Schema

A functor from Schema ⟶ tables or Set is an instanse of a database

Natural transformations
next time